Lecture 3:
Hash indexing, index selection

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Exercise left from last week

- *B^+-trees*
  - Questions a), b), and c).

Today's lecture

- Morning session: Hashing
  - Static hashing, hash functions
  - Extendible hashing
  - Linear hashing
  - Newer techniques:
    Buffering, two-choice hashing
- Afternoon session: Index selection
  - Factors relevant for choice of indexes
  - Rules of thumb; examples and counterexamples
  - Exercises

What data in index?

- At least three possibilities:
  1) Record of key (only primary index)
  2) Key and pointer to record of key.
  3) Key and list of pointers to the records containing the key (for non-unique keys).

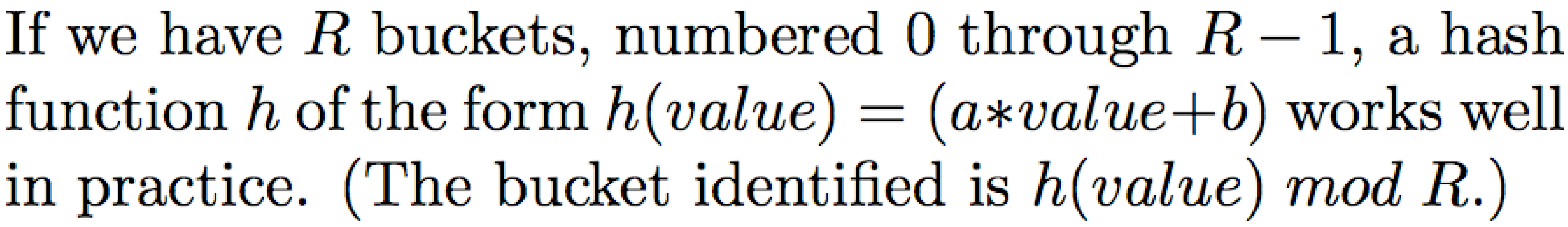
- For simplicity, we consider the case where there is the same number of keys (B) in every disk block.
  - Case 1 with fixed length records.
  - Case 2 with fixed length keys.

Static external hashing

- Hash table:
  - Array of **R** disk blocks (≠notation in RG.)
  - Can access block **i** in 1 I/O, for any **i**.
- Hash function **h**:
  - Maps keys to {0,...,**R**-1}.
  - Should be efficient to evaluate (0 I/Os).
  - Idea: **x** is stored in block **h(x)**.
- Problem:
  - *Dealing with overflows.*
  - Standard solution: Overflow chains.

Problem session

- Consider the following claim from RG:



If we have R buckets, numbered 0 through $R-1$, a hash function h of the form $h(value) = (a*value+b)$ works well in practice. (The bucket identified is $h(value)\ mod\ R$.)

- Donald Ummy uses this hash function in an application, and finds out that it performs terribly, no matter how the constants a and b are chosen.
- What might have gone wrong?

Randomized hash functions

Another approach (not mentioned in RG):

- Choose **h** at random from some set of functions.
- This can make the hashing scheme behave well regardless of the key set.
- E.g., "universal hashing" makes chained hashing perform well (in theory and practice).
- Details out of scope for this course...

Analysis, static hashing

- Notation:
  - R blocks in hash table
  - Each block in the hash table can hold B keys.
- Suppose that we insert $N=\alpha R$ keys in the hash table (”fraction α full”, “load factor α”).
- Assume **h** is truly random.
- Expected number of overflow blocks: $(1-\alpha)^{-2} \cdot 2^{-\Omega(B)} R$ (proof omitted!)
- Good to have many keys in each bucket (an advantage of secondary indexes that store only pointers to records).
- Should keep α away from 1. (How?)

Sometimes, life is easy

- If B is sufficiently large compared to N, all overflow blocks can be kept in internal memory.
- Lookup in 1 I/O.
- Update in 2 I/Os.

Too many overflow chains?

Can have too many overflow chains if:

- The hash function does not distribute the set of keys well (”skew”).
  - Solution 1: Choose a new hash function.
  - Solution 2?: Overflow in main memory.
- The number of keys in the dictionary exceeds the capacity of the hash table.
  - Solution: Rehash to a larger hash table.
  - Better solution: ?
- There are many duplicate values.
  - No fix needed.

Doubling the hash table

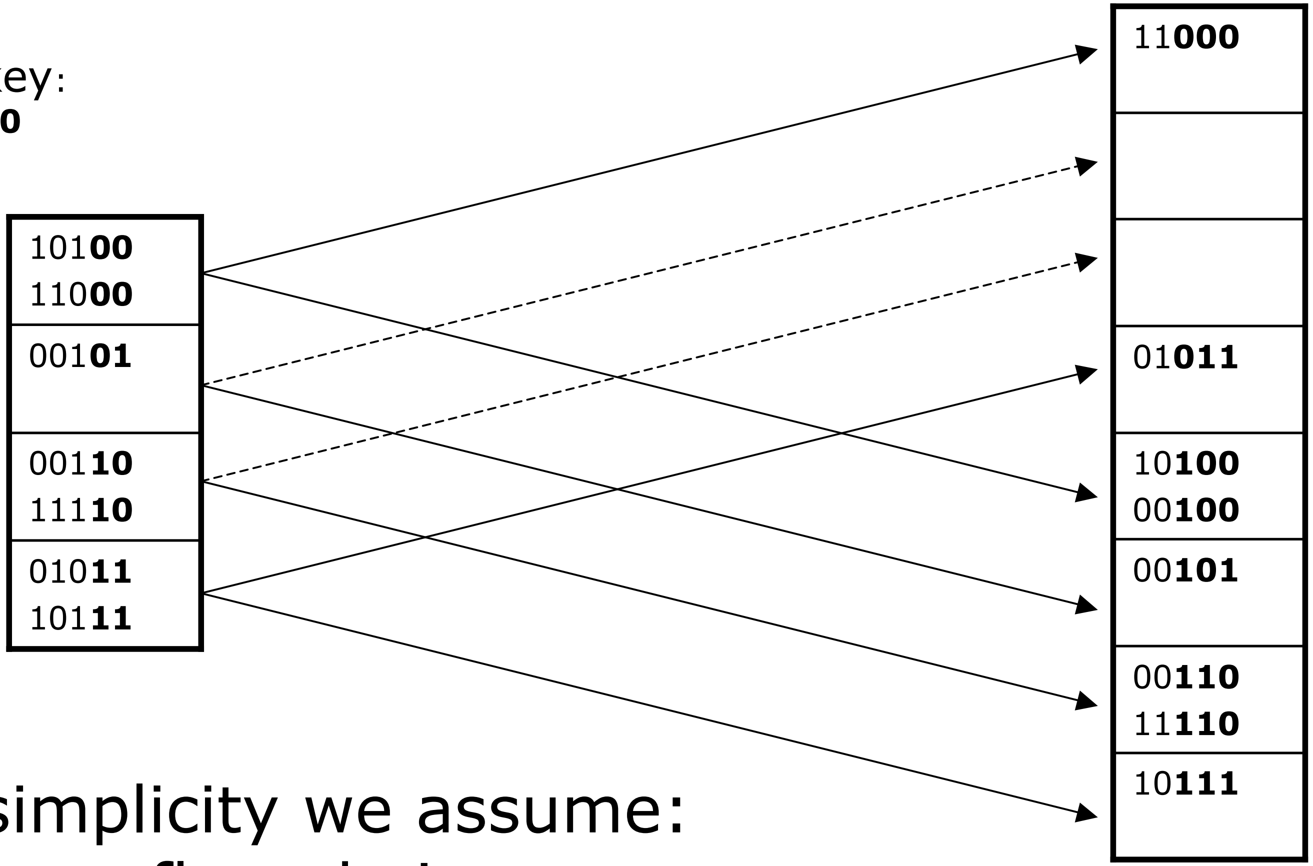
- For simplicity, assume **R** is a power of 2. Suppose **h** is a hash function that has values of ”many” (e.g. 64) bits.
- We can map a key **x** to {0**,...,R**-1} by taking the log **R** least significant bits of **h(x)**.
- Suppose that the hash table has become too small:
  - Want to double the size of the hash table.
  - Just consider one more bit of **h(x)**.

Doubling the hash table, cont.

- Suppose **h(x)**=0111001 (in binary) and the hash table has size 16.
- Then **x** is stored in block number 1001 (binary).
- After doubling to size 32, **x** should be stored in block 11001.
- Generally, all keys in block 1001 should be moved to block 01001 or 11001.
- **Conclusion**: Can rehash by scanning the table and split each block into two blocks.

Doubling, example

New key:
001**00**



For simplicity we assume:

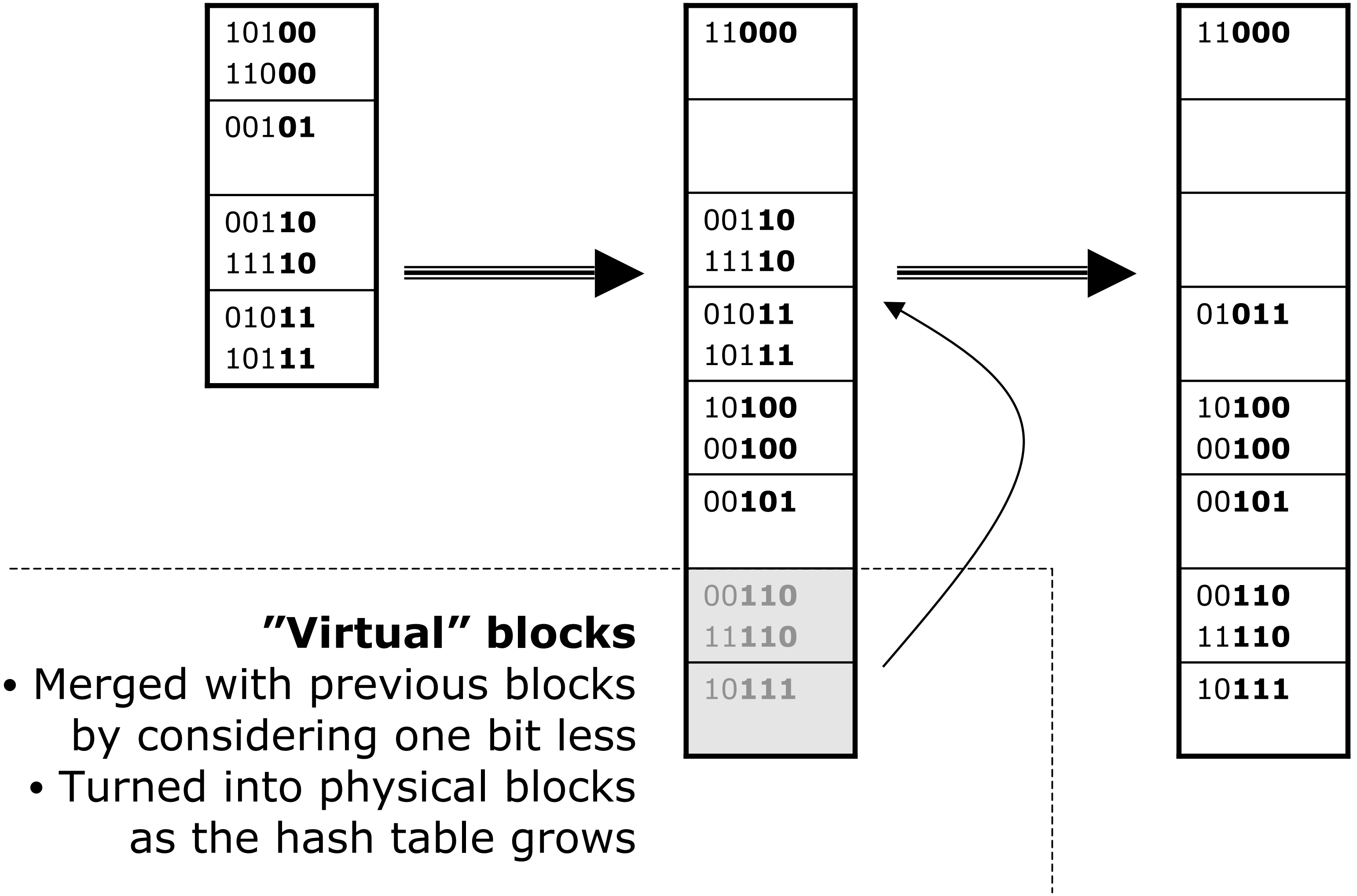
- No overflow chains
- h(x)=x

Problem session

- Find some possible disadvantages of the doubling strategy. Consider:
  - Space usage vs overflows
  - System response time

- **Next**: Alternatives that address some of the disadvantages of doubling.

Linear hashing



101**00**
110**00**

001**01**

001**10**
111**10**

010**11**
101**11**

11**000**

00**110**
11**110**

010**11**
101**11**

10**100**
00**100**

00**101**

00**110**
11**110**

10**111**

11**000**

01**011**

10**100**
00**100**

00**101**

00**110**
11**110**

10**111**

"Virtual" blocks

- Merged with previous blocks by considering one bit less
- Turned into physical blocks as the hash table grows

Linear hashing - performance

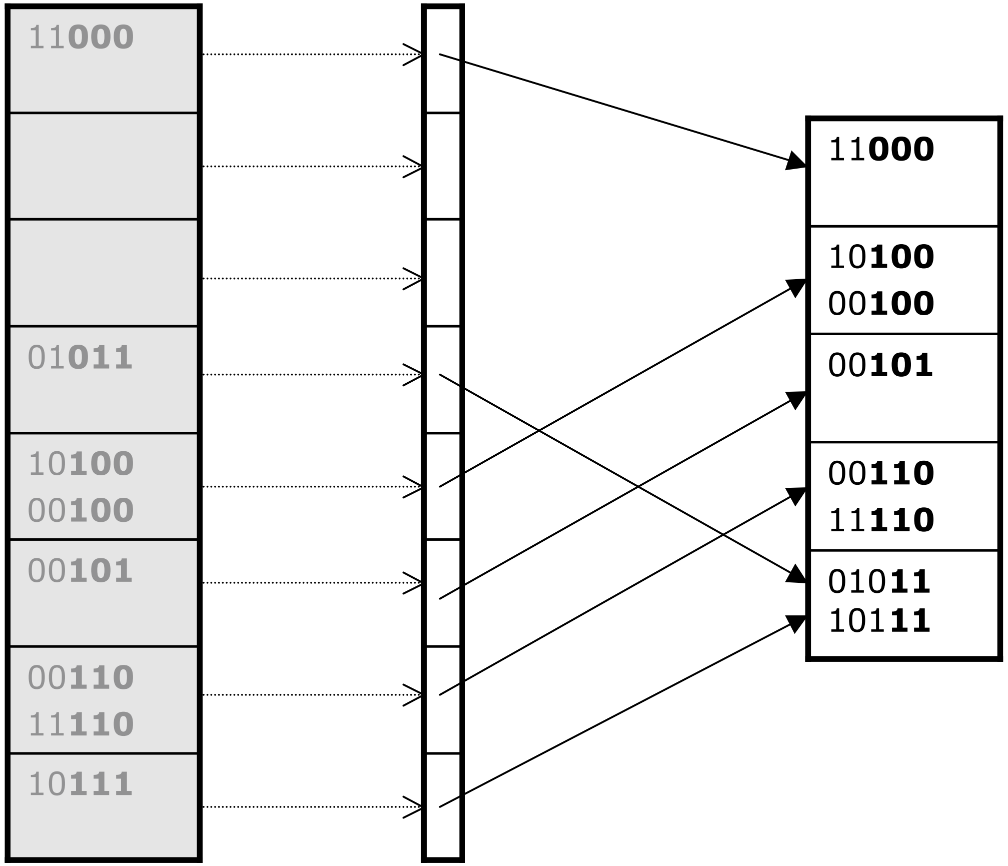
The good:

- Resizes hash table one block at a time: Split a block or merge two blocks.
- Cost of resize: 3 I/Os. Cheap!

The bad:

- Increasing size of hash table may not eliminate any overflow chain.
- Uneven distribution of hash values; works best for relatively low load factors, 50-80%. (But variants of linear hashing improve this.)
- No worst-case guarantee on query time.

Extendible hashing



"Virtual" hash table - no overflows

physical hash table

"Directory" - mapping virtual to physical

Extendible hashing invariants

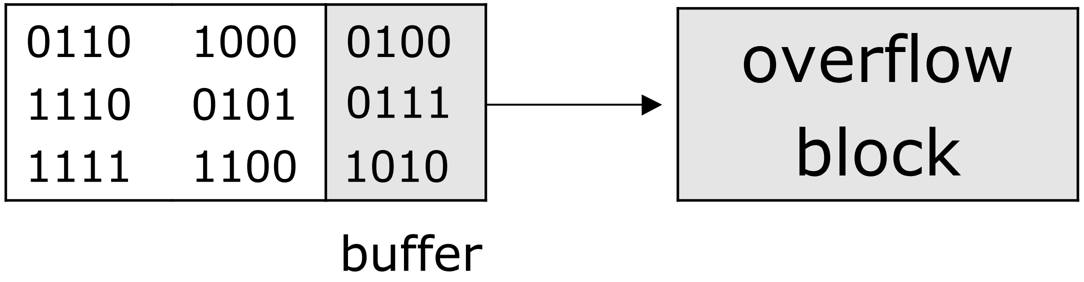
- Virtual hash table has no overflows - may need to increase in size.
- Physical hash table has no overflows.
- Virtual hash table is as small as possible - may need to shrink.
- **”Compression”**: For any bit string **s**, if we consider the virtual hash table blocks whose index ends with **s** then either:
  - These blocks contain more than B keys, or
  - The corresponding entries in the directory all point to the same block. (In other words, these blocks are merged.)

Extendible hashing performance

- At most 2 I/Os for every lookup.
- Only 1 I/O if directory fits in internal memory.
- Space utilization in physical hash table is 69% (expected).
- Size of directory is roughly $\frac{4N}{B}\ \sqrt[B]{N}$ (expected) - this is much smaller than the hash table if B is moderately large.
- Solution with better space usage (if sufficient internal memory):
  B-tree with only leaves on disk.

Buffering

- Same trick as in buffered B-trees: Don’t do updates right away, but put them in a buffer.



0110 1000 0100
1110 0101 0111
1111 1100 1010

buffer

overflow block

- Advantage: Several keys moved to the overflow block at once.
- Disadvantage: Buffer takes space.
- Details in [JensenPagh07].

Two-choice hashing

- **Idea**:
  - Use two hash functions, h_1 and h_2.
  - x is stored in either block $h_1(x)$ or $h_2(x)$, use two I/Os for lookup.
  - When inserting x, choose the least loaded block among $h_1(x)$and $h_2(x)$.
- Can be shown that overflow probabilities are much smaller than with one function, especially when B is small.
- If two disks are available, the 2 I/Os can be done in parallel.

Today’s lecture, part 2

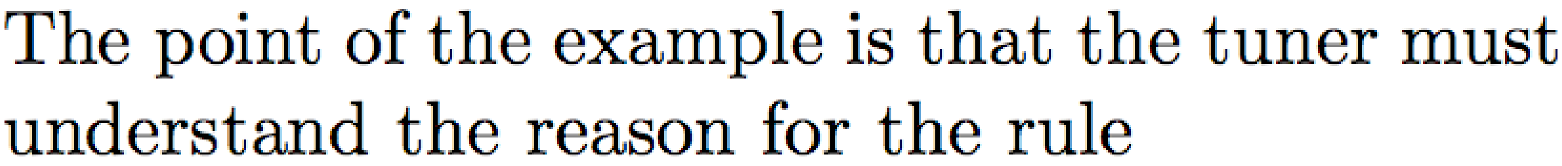
- Index selection
  - Factors relevant for choice of indexes
  - Rules of thumb; examples and counterexamples
- Exercises

Workload

- The workload (mix of operations to be carried out by the DBMS) has a large influence on what indexes should be created in a database.
- Other factors are:
  - the data in relations, and
  - the query plans produced by the DBMS.

Rules of thumb

- Rules of thumb can be used to guide thinking, and as a checklist.
- Are often valid in most cases, but there are always important exceptions.
- Quote from SB:



> The point of the example is that the tuner must understand the reason for the rule

- You don’t yet have the entire picture (query optimization, concurrency), but we can start reasoning about rules anyway.

Rule of thumb 1:

Index the most selective attribute

- Argument: Using an index on a selective attribute will help reducing the amount of data to consider.
- Example:

```
SELECT count(*) FROM R
WHERE a>'UXS' AND b BETWEEN 100 AND 200
```

- Counterexamples:
  - Full table scan may be faster than an index
  - It may not be possible/best to apply an index.

Rule of thumb 2:

Cluster the most important index of a relation

- Argument:
  - Range and multipoint queries are faster.
  - Usually sparse, uses less space.
- Counterexamples:
  - May be slower on queries ”covered” by a dense index. (More on this later.)
  - If there are many updates, the cost of maintaining the clustering may be high.
  - Clustering does not help for point queries.
  - Can cluster according to *several* attributes by duplicating the relation!

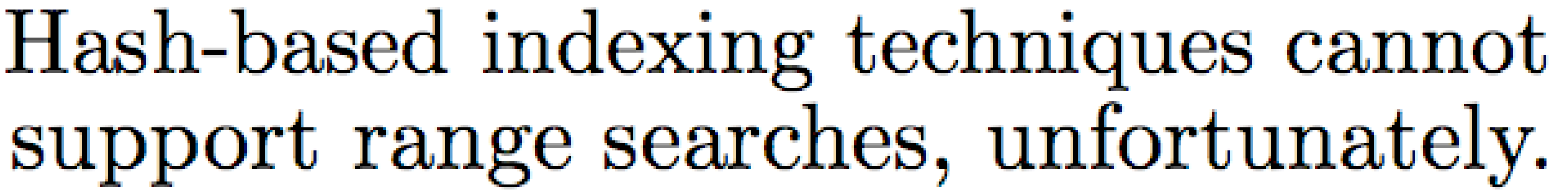
Rule of thumb 3:

Prefer a hash index over a B-tree if point queries are more important than range queries

- Argument:
  - Hash index uses fewer I/Os per operation than a B-tree.
  - Joins, especially, can create many point queries.
- Counterexamples:
  - If a real-time guarantee is needed, hashing can be a bad choice.
  - Might be best to have **both** a B-tree and a hash index.

Aside: Hashing and range queries

RG page 371:



Hash-based indexing techniques cannot support range searches, unfortunately.

- **But**: they can be *used* to answer range queries in **O(1+Z/B)** I/Os, where Z is the number of results. (Alstrup, Brodal, Rauhe, 2001; Mortensen, Pagh, Patrascu 2005)
- Theoretical result on external memory (why?) - and out of scope for DBT.

Problem session

- Setting:
  - we have 2^{20} tuples in a primary index
  - tuples take the space of 4 keys,
  - the space for a pointer is small compared to the space of a key
  - internal memory has space for $M=2^{16}$ keys.
- Consider the search time of B-trees and extendible hashing two cases:
  - Case A: $B=4$ (i.e., 4 tuples/block).
  - Case B: $B=2^6$.

Rule of thumb 4:

Balance the increased cost of updating with the decreased cost of searching

- Argument: The savings provided by an index should be bigger than the cost.
- Counterexample:
  - If updates come when the system has excess capacity, we might be willing to work harder to have indexes at the peaks.
- If buffered B-trees are used, the cost per update of maintaining an index may be rather low. Especially if small degree trees are used.

Rule of thumb 5:

A non-clustering index helps when the number of rows to retrieve is smaller than the number of blocks in the relation.

- Argument:In this case it surely reduces I/O cost.
- Counterexample:
  - Even for a non-clustered index, the rows to retrieve can sometimes be found in a small fraction of the blocks (e.g. salary, clustered on date of employment).

Rule of thumb 6:
Avoid indexing of small tables.

- Argument: Small tables can be kept in internal memory, or read entirely in 1 or 2 I/Os.
- Counterexample:
  - If the index is in main memory, it might still give a speedup.

Rule of thumb 7:

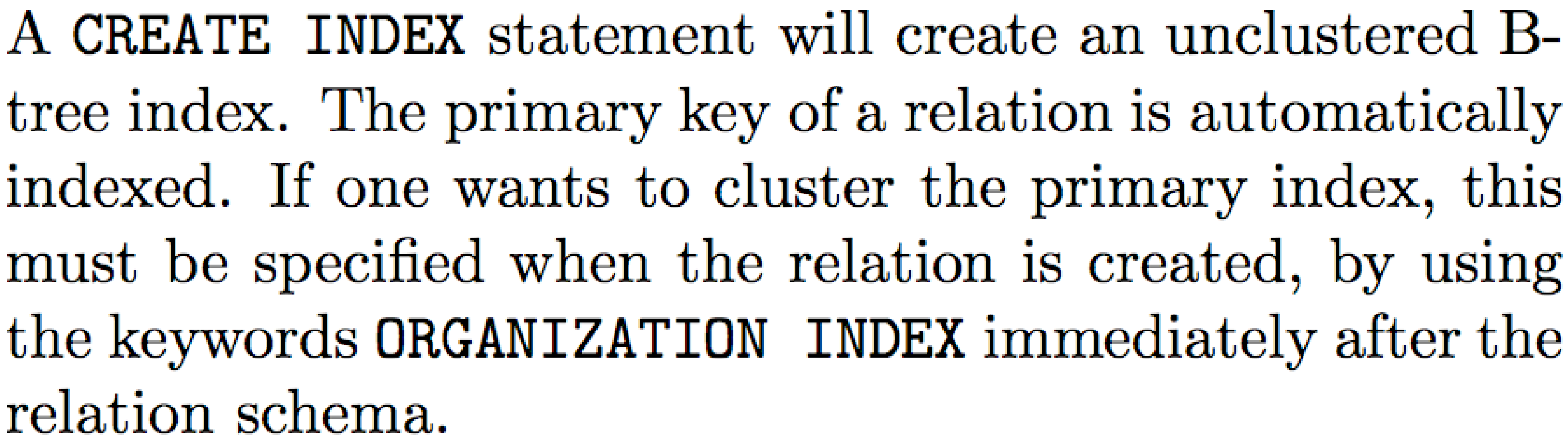
A *covering index* for a query will speed it up

- Argument: The index will contain less data than the base table, allowing a faster scan of all data needed.
- Counterexamples:
  - If the table is vertically partitioned, a similar speedup can be achieved.
  - A vertically partitioned relation may have *several* indexes that can be used to answer the query (e.g. an index to select and an index to join).

Conclusion

- Indexing is a complicated business!
- Understanding the various index types and their performance characteristics, as well as the characteristics of the database at hand and its workload allows informed indexing decisions.
- Rules of thumb can be used to guide thinking.
- More complications to come!

Tip: Clustered indexing in Oracle



A `CREATE INDEX` statement will create an unclustered B-tree index. The primary key of a relation is automatically indexed. If one wants to cluster the primary index, this must be specified when the relation is created, by using the keywords `ORGANIZATION INDEX` immediately after the relation schema.

- To cluster according to a *non-unique* attribute A, declare a composite primary key (A,P), where P is a unique key.

Tip: Hash indexing in Oracle

It is also possible organize a table in a clustered hash index. First, create the hash table (called a "cluster") using

```
CREATE CLUSTER <hashtablename>(<key>)
HASHKEYS <hashtablesize>,
```

and then, when creating the table, specify that is to be stored in the hash table:

```
CREATE TABLE <tablename>(<schema>)
CLUSTER <hashtablename>(<key>).
```

The size of the hash table is fixed, i.e., the hash table will not grow or shrink

Exercises

Hand-outs:

- *Choosing an index.*
  - Questions a), b), and c).

- *Representation of relations*
  - Question d

  (on handout from last week).