Advanced Database Technology

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Text indexing

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Based on
[Kärkkäinen-Rao03]

Text indexing

Data in form of text:

- Digital libraries
- WWW, HTML-pages
- Biological data (e.g. DNA)
- XML
- ...

We want to:

- Search for specific strings
- Compare strings
- Analyze text data

Problem session

Can relational databases be used for text data? Think about it by considering the following two situations:

- We have a number of HTML-pages for which we want a database so that we can search for all pages containing a specific key-word. How can the HTML-pages be stored in a relational database and what query will return the desired answer?
- We have a collection of DNA strings and we want a database that supports queries of the form "Is ATTG...T a substring of any of the DNA stings in the database?". How can this be implemented using a relational database?

This lecture

- Indexed string matching problem
- Internal memory data structures
  - Suffix array
  - Suffix tree
- External memory data structures
  - Patricia trie and Pat tree
  - Short Pat array
  - String B-tree

Terminology

An alphabet, denoted Σ, is a set of (ordered) characters.

A string S is an array of characters, $S[1,n] = S[1]S[2]\cdots S[n]$.
$S[i,j] = S[i]\cdots S[j]$ is a substring of S.

$S[1,j]$ is a prefix of S. $S[i,n]$ is a suffix of S.

Σ^* denotes all strings over alphabet Σ.

We can sort strings according to lexicographic order.
Example: For $\Sigma = \{a, b, c, \ldots, z\}$, where $a < b < c < \cdots < z$, the lexicographic order is the same as in a dictionary.

Problem definition

Indexed String Matching Problem

Let T be a set of K strings in Σ^*, where N is the total length of all strings in T.

String matching query on T: Given a pattern P find all occurrences of P in the strings in T.

Static problem: Store T in a data structure that supports string matching queries. Such a data structure is called a full-text index.

Dynamic version: Supports also insertions and deletions of strings in the full-text index.

Suffixes and prefixes

An important observation:

For all occurrences of a pattern P in a text T there is a suffix of T that has a prefix equal to P
(i.e., any substring of T can be written as a prefix of a suffix of T)

Suffix array

A suffix array of a text T is denoted SA_T.

An array with one entry for each suffix in T.

The suffixes are sorted. Pointers to the suffixes of T are stored (in sorted order) in the array.

Note that all suffixes with the same prefix are stored in consecutive order in the array.

Searching in a suffix array:

Binary search. $O(\log N)$ string comparisons in at most $O(|P| \log N)$ time.

Possible to do it faster: $O(|P| + \log N)$ [Manber and Myers].

Construction:

In time $O(N \log N)$.

Trie

- A rooted tree with edges labeled by characters.
- A node in the tree represents the string spelled out by following the path from the root to that node.
- A trie for a set T of strings is the minimal trie, such that for all strings $t \in T$ there is a node in the trie representing t.

Compact trie: Trie where paths without branches are replaced with one edge labeled with the string labeling the path.

Suffix trees

Data structure for strings with many applications.

For example:
Given a string S preprocess S such that the query

- Is P a substring of S?

for any string P can be answered in time proportional to $|P|$ and not $|S|$.
Suffix trees: $O(|S|)$ time preprocessing and $O(|P|)$ time for queries.

Suffix trees

A suffix tree for a string S of length m is a rooted tree with m leaves labeled $1, \ldots, m$, with the following properties:

- Internal nodes have degree 2 or more (except possibly the root).
- Edges are labeled with nonempty substrings of S.
- Two edges out of the same node are not labeled by the same first character.
- The concatenation of the edge labels on the root-to-leaf path to leaf i spells out the suffix starting at position i, i.e., $S[i..m]$.

Suffix trees in external memory

To get linear space for the suffix tree, the edge labels can not be stored in the tree. Instead pointers to the string are stored. This is a problem in external memory.

Problem session

- How much space would be needed if the strings were stored along with the edges in the suffix tree?
- Why is it a problem to use pointers to the string when we think about external memory?

Patricia trie and Pat tree

A Patricia trie is a compact trie, except that the edges are labeled in an other way.

- An edge is labeled by only the first character of the label in the compact trie and the length of the string labeling the edge in the compact trie.
- In the leaves there are pointers to the strings.

A Pat tree for a text T is denoted PT_T. It is the Patricia trie for the set of suffixes in T.

Searching in a Pat tree

How to search:

Search is almost like in the suffix tree.

– When searching for pattern P, compare the first character in P with the characters labeling the edges to the children of the root. Skip the following $x-1$ characters where x is the number labeling the edge with the same character as P. Continue recursively.

– When reaching the end of the pattern P we know that all leaves in that subtree have the same prefix. Look at one of them and compare to P. If it matches P, then all match P, otherwise none match P.

Searching in a Pat tree

Idea:

When searching in the Pat tree you do not have to access the text more than once. Therefore it is (often) much better for external memory.

Time to search:

$O(|P| + Z)$, where Z is the size of the answer.

Short Pat array (SPat array)

External memory version of a suffix array.

Idea:

- Divide the suffix array into buckets of equal size, say size p.
- Take one (the last) string in each bucket, and form a new shorter array.
- Store only the first l characters of each string in the array.
- Choose the bucket size p and length l so that this SPat array fits in memory.
- (More levels can be used if the buckets are too large to fit in one block on disk. Assume for now that p is not larger than the block size.)

Searching a SPat array

The SPat array can be searched (using, e.g., binary search) without extra I/O's.

- If the pattern P matches more than one string in the SPat array in the first l characters, then we need to look at the actual strings. Let r be the number of strings in the SPat array matching P.
  $O(\log r)$ I/O's are needed to find the bucket(s) to search in.
- A search in a bucket is done by performing a binary search among the p strings.
  $O(\log p)$ I/O's are needed to find the left and right end points.
  (assuming each string comparison only takes $O(1)$ I/Os).

Problem session

What are the two main problems with using a B-tree for storing strings of variable length?

String B-trees

String B-trees combines B-trees and Patricia tries to overcome the problems with comparing strings and storing strings in the nodes of a B-tree.

Definition:

- Each nodes in the string B-tree stores a Patricia trie for the set of strings consisting of the lexicographic smallest and largest strings in all its subtrees.
- The degree of a node is between $b/2$ and b (except for the root).
- The strings in the leaves are lexicographically sorted from left to right.

Searching in String B-trees

The essential step is to use the Patricia trie to find the right subtree to continue the search in.

First, match the pattern in the Patricia trie to reach a leaf (if the pattern mismatches or is exhausted at an internal node, take any leaf in its subtree).

Note: The pattern P may have a mismatch before the mismatch in the Patricia trie, since only the first character after a branching point is compared to P.

Compare P to the string pointed to by the leaf. From this, we can find the "predecessor" of P in the Patricia trie. We know where to continue to search (or that P doesn't match any string in the tree).

Query time: $O(scan(|P| + Z) + search(N))$
where $scan(X) = \Theta(X/B)$ and $search(N) = \Theta(\log_B N)$.

Updates in String B-trees

Insertion:

Similar to insertion in B-trees, but now we must insert new strings into the Patricia tries in the nodes.

Insertion of a string S can be done in $O(scan(|S|) + search(N))$ I/O's, where N is the number of strings in the String B-tree.

The naive way of doing it takes $O(scan(|S|))$ I/O's in each node that is changed.

$O(scan(|S|) + search(N))$ is also the amortized cost using the naive algorithm, since the amortized number of split nodes is constant.

Deletion: Similar to insertion.

Summary of external text indexes

Patricia trie and Pat tree:

$O(|P| + Z)$ I/O's (naive implementation).

Efficient if the tree fits in main memory, i.e. $Scan(|P|)$ I/O's.

Used in String B-trees.

Short Pat array:

Efficient if enough memory to store the SPat array and if the SPat array does not contain too many strings with equal first l characters.

String B-tree:

Searching in $O(scan(|S| + Z) + search(N))$ I/O's.

Updates in $O(scan(|S|) + search(N))$ I/O's.

Space needed: $\Theta(N/B)$ blocks.

Can also be used for dynamic indexed string matching.