Advanced Database Technology
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Geometric index structures

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Based on GUW Chapter 14.0-14.3, [Arge01] Sections 1, 2.1 (persistent B-trees), 3-4 (static versions only), 4.1, 9.

Today's lecture

- Multidimensional data
- Commonly used (heuristic) geometric index structures
- Persistent B-trees
- Stabbing query problem
- Planar point location
- The logarithmic method

Multidimensional data

Geometric data is one, two, or three dimensional, but also all relational data can be seen as multidimensional data.

- A relation with k attributes can be seen as a k-dimensional space.
- A tuple can be seen as a point in the k-dimensional space. The coordinates for the point are the values for the attributes.

Queries on geometric data

- **Partial match queries**: look up all points matching one or more attributes.
- **Range queries**: look up all points within a range for one or more attributes.
- **Nearest-neighbor**: find the point nearest to a query point.
- **Where-am-I**: given data in the form of geometric objects, find the objects that intersect the query point.

2-d range query in SQL

Given the relation:

Student(Name, age, enrolled_year)

Find all students aged 30 to 40 who started to study before 2004.

```
SELECT Name
FROM Student
WHERE age<=30 AND age>=40 and
  enrolled_year<2004;
```

Geometric index structures

Hash-like indexes

- Grid files
- Partitioned hash functions

Tree-based indexes

- Multiple-key indexes
- kd-trees
- Quad trees
- R-trees

These indexes are *heuristic*, i.e., they are not (proved to be) worst case efficient.

Grid files

- Simple idea:
  - Split each dimension into intervals.
  - Put each point into a bucket corresponding to the intervals it lies in (Ex: GUW p. 677).
- Overflow handling as in hash tables.
- Supports partial match queries, range queries, and nearest neighbor queries (how?)
- Bad case: All data along ”diagonal” - need many grid lines (in internal memory) to avoid large buckets.

Multiple-key index

- Index for several attributes A, B, C,…:
  - Group index attributes as a tuple (A,B,C,…)
  - Order among tuples is **lexicographic**.
  - Make B-tree index according to this order.
  - (Note: Different exposition in GUW).
- Efficiently supports partial match queries on a **prefix** of the attributes (corresponds to a range query).
- Bad case: Partial match query on non-prefix, e.g., search for single value of last attribute.

kd-trees

- Short for ”k-dimensional search tree”.
- Change from ordinary search trees:
  - Each node is associated with a dimension.
  - An internal node partitions the points of its subtree along its dimension.
  - Dimensions rotate down the tree: 1,2,..,k,1,2,..k,1,2,… (Ex: GUW p. 691)
- Supports partial match queries, range queries and nearest neighbor queries.
- Bad case: Many points with same value in some dimension makes it impossible to split points well along this dimension.

R-trees

- In a B-tree each interior node:
  - Corresponds to a 1-dimensional range.
  - ”Knows” the ranges of its children.
- In an **R-tree** each interior node:
  - Corresponds to a k-dimensional rectangle.
  - ”Knows” the rectangles of its children.
- Supports partial match queries, range queries and nearest neighbor queries,…
- Flexible: All kinds of geometric objects (not just points) fit into rectangles.
- Unspecified (and hard): Maintaining ”good” rectangles.

Persistent data structure

- A persistent data structure supports queries on previous versions of the data structure.
- A query specifies a time, e.g., “Was element x in the data structure at time t”?
- Updates are only supported at current time, not in an earlier version.
- Possible solution: Copy the data structure when it is updated. (Inefficient!)
- Similar to the concept of *temporal databases.*

Persistent B-trees

- One data structure representing **all versions** of the B-tree.
- Elements have an ***existence interval***: it exists from the time of insertion until time of deletion (or until now if it is still in the current version).
- Nodes in the B-tree also have an ***existence interval***.
- Nodes and elements are ***alive*** in their existence intervals.
- **Invariant**: A node contains $\Theta(B)$ alive children in its existence interval.
- **Note**: Nodes alive at time t make up a B-tree for elements alive at time t.

Searching and updates in persistent B-trees

- **Searching for x at time t** can be done as usually in time $O(\log_B N)$ in the tree consisting of nodes alive at time t.

- **Insertion of x** is similar to normal insertion in a B-tree. If x should be inserted in leaf l, and l is full, then we have to maintain the new invariant. (Blackboard)

- **Deletion:** The element is not deleted, but the time interval is updated. This may cause a violation of the invariant. (Read yourself)

Time and space for persistent B-trees

- **Construction:** Insert elements one by one: N insertions take $O(N \log_B N)$ I/Os.
- In fact, construction can be done in $O(N/B \log_{M/B}(N/B))$ I/Os.

- **Space:** $O(N/B)$ blocks.

- **Note**: N is the total number of elements, both alive and deleted elements.

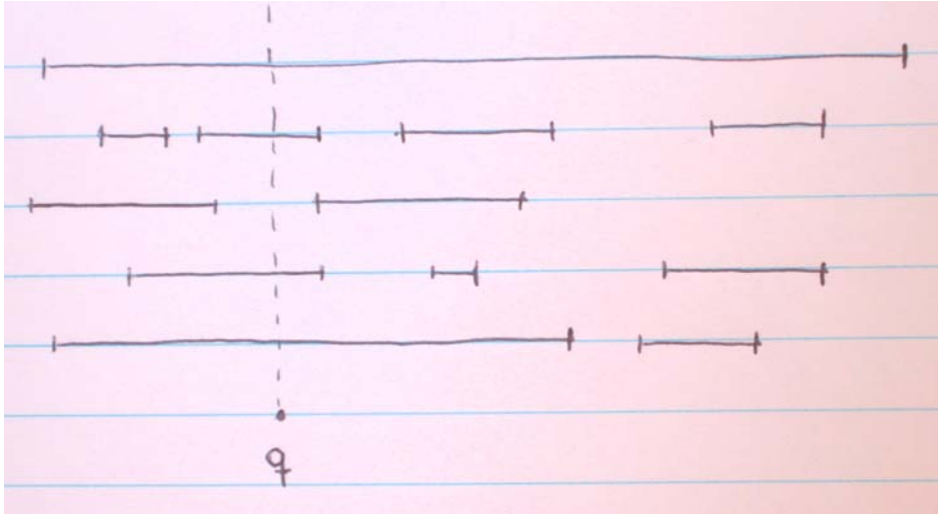
Problem session

Why are we talking about persistent B-trees in a lecture on geometric data?

- How can we use a persistent B-tree to represent 2-d (geometric) data?
- Given a set of points in 2-d, how can we perform a 3-sided, 2-d range query using the persistent B-tree?

Stabbing query problem

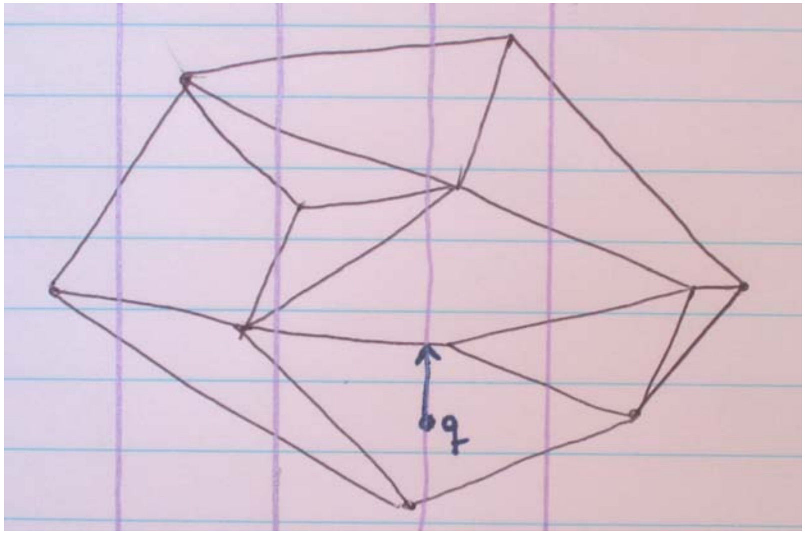
- Data structure for a set of intervals (1-d).
- Query: Report all intervals containing point q.
- Static version.
- Use *the logarithmic method* to get a dynamic data structure supporting insertions of intervals.



Stabbing query problem (cont.)

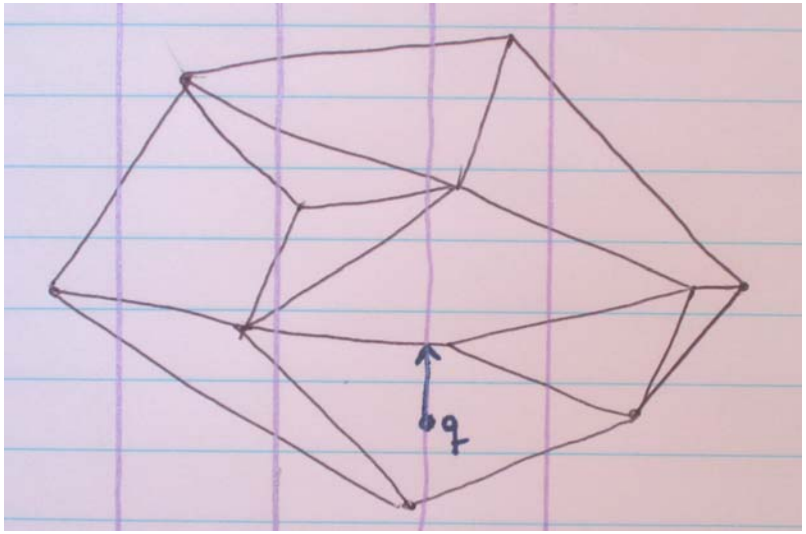
- Use a persistent B-tree with intervals as elements and interval endpoints as times.
- *Sweep* the intervals from left to right.
- Insert an interval when the sweep line reaches its left endpoint.
- Delete an interval when the sweep line reaches its right endpoint.
- **Construction time:**
  $O(N/B \log_{M/B}(N/B))$
- **Query:** Report all elements alive at time q. $O(\log_B N+T/B)$. (T=output size)

Planar point location



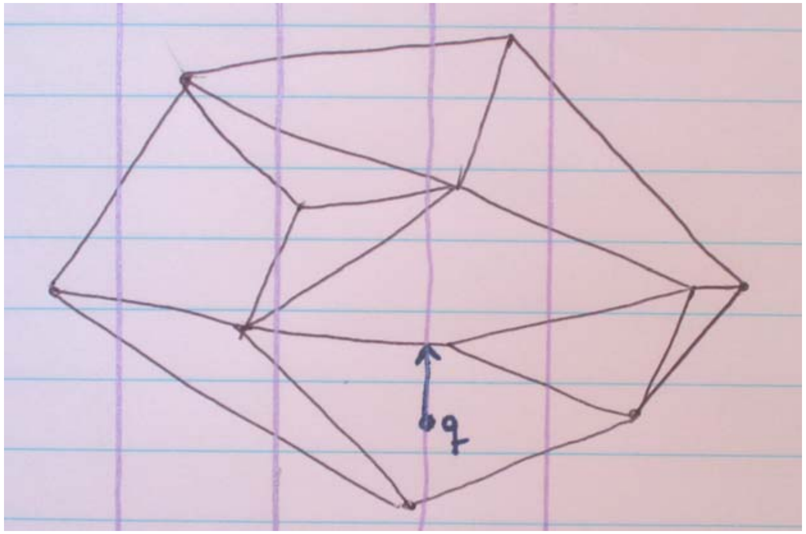
- Given a planar subdivision with N vertices.
- We want a data structure supporting:
  - **Query**: "Which region contains point q=(x,y)?"
- Assume: Enough to find one segment of the region (the one straight above q).

Planar point location (cont.)



- Idea: Use a persistent B-tree.
- Segments are elements.
- A segment exists in the time interval from left x-coordinate to right x-coordinate.
- Search at time x (q=(x,y)). How do we find the right segment?

Problem session



- Segments can not be ordered (in a given direction) in general. Why not?
- We need an order of the segments to search in the B-tree. Which segments do we need to compare?
- How can it be done?

Planar point location (cont.)

- Search for ”segment” q=(x,y), in the persistent B-tree at time x.
- A point can be compared to a segment.

- Search time: $O(\log_B N)$ I/Os.
- Construction time: $O(N \log_B N)$ I/Os.
- Space: linear ($O(N/B)$ blocks).

The logarithmic method

- A general method to make many static data structures dynamic.
- Internal memory version:
  - Partition the N elements into log N sets of size $2^0, 2^1, 2^2, \ldots$
  - Build a static data structure for each set, denoted $D_0, D_1, D_2, \ldots$
  - Every query has to query each of the sets.
  - Insertion: Find first empty D_i. Build the structure D_i with all elements in the D_j's for $j<i$ and the new element.
  - Note: $2^0+2^1+\ldots+2^{i-1}=2^i-1$
  - Amortized cost: x is in at most $\log_2 N$ d.s.

The logarithmic method - external memory

- $\log_B N$ subsets
- D_i has size at most B^i

- **Query**: Query all structures.
- **Insertion**: Insert in smallest D_i where $|D_1|+|D_2|+\ldots+|D_i|<B^i$. The new data structure for D_i contains all elements in D_j, $j \leq i$, and the new element.
- **Deletion**: Mark elements deleted.

The logarithmic method - external memory (cont.)

Analysis:

- Assume a static structure with construction cost $T(N)$ and query cost $Q(N)$.
- **Query time**: $O(\sum^{\log_B N} Q(|D_i|))$. If $Q(N)=O(\log_B N)$ then the query time is $O(\log_B^2 N)$.
- **Amortized cost of insertion**:
  - Note 1: D_i may be smaller than B^i, and it is rebuilt more than once. Hence an element may be in D_i during several rebuilds.
  - Note 2: At least B^{i-1} new elements in D_i each rebuild.
  - Note 3: An element never moves ”down”.
  - If $T(N)=O(N/B \log_B N)$, then insertion costs $O(\log_B^2 N)$ I/Os, amortized.